Name______________________ Teacher________________

GOSFORD HIGH SCHOOL

2014

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK 2

MATHEMATICS

Duration- 90 minutes plus 5 minutes reading time

Section 1 **Multiple choice**	**5** questions worth **1 mark each.** (Answer this section on the **multiple choice response sheet provided)**	**/5**
Section 2 **Question 6**	Integration	**/15**
Question 7	Exponentials and Logarithms	**/15**
Question 8	Preliminary topics Applications of calculus	**/8** **/7**
TOTAL		**/50**

HSC MATHEMATICS

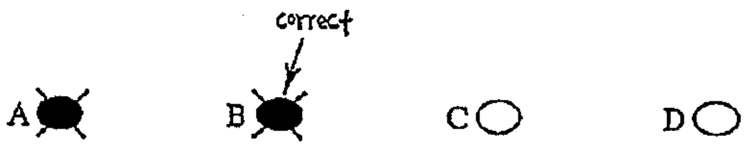
Student Name/Number:_________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

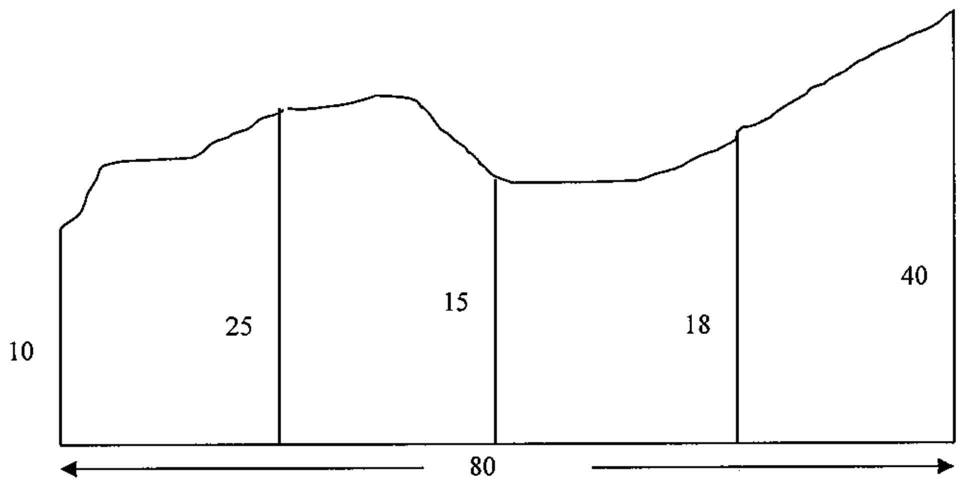
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

SECTION 1: MULTIPLE CHOICE. Questions are worth 1 mark each. Answer on the multiple choice answer sheet provided.

Question 1.



The field drawn is to have its area approximated by applying the trapezoidal Rule. The value of the area is:

(A) 6640 square units.   **(B)** $110\frac{2}{3}$ square units.

(C) 1660 square units.   **(D)** 1680 square units.

Question 2.

The solution to the equation $2log_5 3 = log_5 x - log_5 6$ is:

(A) $1\frac{1}{2}$ **(B)** 54

(C) 12   **(D)** 36

Question 3.

Given that $\frac{d}{dx}\left(e^{x^2}\right) = 2xe^{x^2}$, then $\int xe^{x^2}dx =$

(A) $\frac{1}{2}xe^{x^2} + c$ **(B)** $2e^{x^2} + c$

(C) $\frac{1}{2}e^{x^2} + c$ **(D)** $2xe^{x^2} + c$

Question 4.

The value of k for which $x^2 - (k-1)x - 2(k+1) = 0$ has a root equal to -1 is:

(A) -2

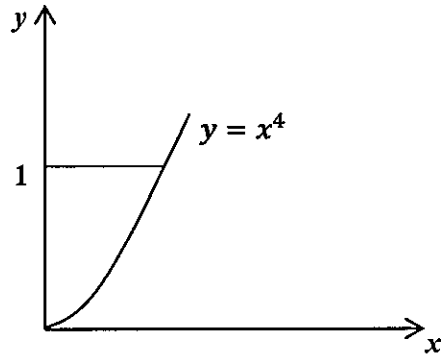
(B) 0

(C) 2

(D) $-\frac{2}{3}$

Question 5.

The region in the diagram bounded by the curve $y = x^4$, the y-axis and the line $y = 1$ is rotated about the x axis.



Which of the following expressions gives the volume of the solid of revolution formed?

(A) $V = \pi \int_0^1 x^8 dx$

(B) $V = \pi \int_0^1 y^{\frac{1}{2}} dx$

(C) $V = \pi \int_0^1 x^4 \, dx$

(D) $V = \pi - \pi \int_0^1 x^8 \, dx$

SECTION 2: Questions are worth 15 marks each. Answer on your own paper. Start each question on a new sheet of paper. All necessary working must be shown.

Question 6

a) Find the following indefinite integrals.

i) $\int 2x^3 + 6x - 1\, dx$ **(1)**

ii) $\int (2 - 3x)^5\, dx$ **(1)**

iii) $\int \frac{(x^2-1)^2}{x^2}\, dx$ **(2)**

iv) $\int \sqrt{\frac{1}{x}}\, dx$ **(2)**

b) Evaluate

i) $\int_1^3 6x^2 + 2x - 1\, dx$ **(2)**

ii) $\int_{-1}^1 x^3(x^2 - 1)^2\, dx$ **(1)**

c)

i) Show that the curve $y = x^3 + x - 2$ is monotonically increasing. **(1)**

ii) Given that the x intercept of the curve is (1,0), calculate the area between the curve and the x axis from $x = 1$ to $x = 2$. **(2)**

d) Find the area between the curve $y = x^2 + 1$ and the straight line $y = x + 7$.

(3)

Question 7 (Start a new page)

a) For the curve $y = log(x - 2)$:

i) write its domain **(1)**

ii) sketch the curve **(1)**

b) Solve for x: $\sqrt[x]{m} = n^3$ **(2)**

c) Find the primitive function of e^{3x+2} **(1)**

d) Find the second derivative of $y = e^{x^2}$ **(3)**

e) Find the volume of revolution when the curve $y = e^x + 1$ is rotated about the x axis from $x = 0$ to $x = 2$. **(2)**

f) Consider the function $y = \frac{1}{x}e^{-x}$

i) For what values of x is the function defined? **(1)**

ii) Find any stationary points and determine their nature. **(3)**

iii) Sketch the curve of this function. **(1)**

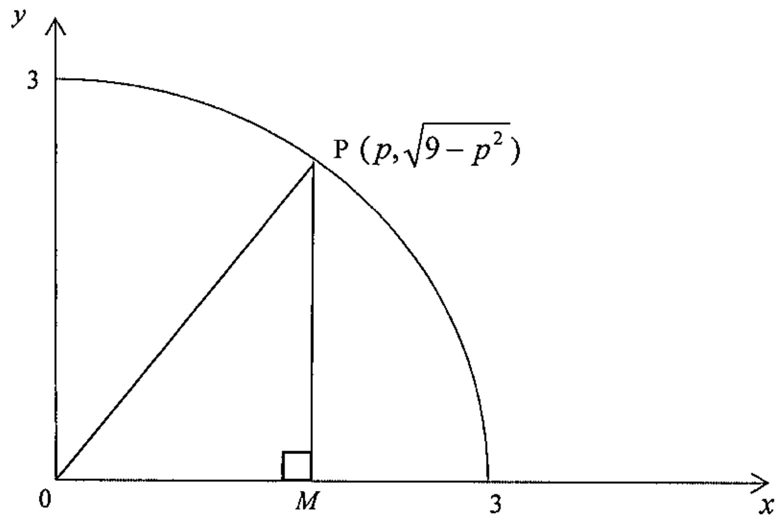
Question 8 (Start a new page)

a) For what value(s) of k is the expression $x^2-(2+k)x+3k+6$ positive definite? **(3)**

b) Find the shortest distance from the point $(8,4)$ to the line $x-2y-1=0$ **(2)**

c) For what values of x is the curve $y=x^3+6x^2-4x+7$ concave down? **(3)**

d)



The diagram shows the curve $y=\sqrt{9-x^2}$ for $x\geq 0$. P is the point $\left(p,\sqrt{9-p^2}\right)$ on the curve and M is the foot of the perpendicular drawn from P to the x axis.

i) Calculate the area bounded by the curve $y=\sqrt{9-x^2}$ and the coordinate axes. **(1)**

ii) Show that the area, A, of the triangle OPM is given by $A=\frac{p\sqrt{9-p^2}}{2}$ **(1)**

iii) Find the coordinates of the point P which gives triangle OPM a maximum area. **(3)**

iv) Show that the ratio of the area of triangle OPM found in part iii) to the area bounded by the curve and the coordinate axes is $1:\pi$. **(2)**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2014 HSC MATHEMATICS
ASSESSMENT TASK 2
SOLUTIONS

SECTION 1

Question 1

$$A = \frac{20}{2}\left[10+40+2(25+15+18)\right]$$
$$= 10[50+116]$$
$$= 1660 \qquad \therefore (C)$$

Question 2

$$\log_5 3^2 = \log_5 \frac{x}{6}$$
$$\frac{x}{6} = 9$$
$$x = 54 \qquad \therefore (B)$$

Question 3

(C)

Question 4

sub $x=-1$

$$1+k-1-2k-2=0$$
$$-k-2=0$$
$$k=-2 \qquad (A)$$

Question 5

(D)

SECTION 2

Question 6

a) i) $\int 2x^3+6x-1\,dx$

$$= \frac{2x^4}{4}+\frac{6x^2}{2}-x+C$$
$$= \frac{x^4}{2}+3x^2-x+C$$

ii) $\int (2-3x)^5\,dx$

$$= \frac{(2-3x)^6}{-18}+C$$

iii) $\int \frac{(x^2-1)^2}{x^2}\,dx$

$$= \int \frac{x^4-2x^2+1}{x^2}\,dx$$
$$= \int x^2-2+\frac{1}{x^2}\,dx$$
$$= \frac{x^3}{3}-2x-\frac{1}{x}+C$$

iv) $\int \sqrt{\frac{1}{x}}\, dx$

$= \int x^{-1/2}\, dx$

$= 2x^{1/2} + c$

$= 2\sqrt{x} + C$

b) i) $\int_1^3 6x^2 + 2x - 1\, dx$

$= \left[2x^3 + x^2 - x\right]_1^3$

$= (54 + 9 - 3) - (2 + 1 - 1)$

$= 58$

ii) $\int_{-1}^{1} x^3(x^2-1)^2\, dx$

$= 0$ (odd function)

c) i) $y = x^3 + x - 2$

$\frac{dy}{dx} = 3x^2 + 1$

+ve for all x

∴ monotonically increasing

ii)

$A = \left|\int_{-1}^{1} x^3 + x - 2\, dx\right| + \int_1^2 x^3 + x - 2\, dx$

$= \left|\left[\frac{x^4}{4} + \frac{x^2}{2} - 2x\right]_{-1}^{1}\right| + \left[\frac{x^4}{4} + \frac{x^2}{2} - 2x\right]_1^2$

$= \left|\left(\frac{1}{4} + \frac{1}{2} - 2\right) - \left(\frac{1}{4} + \frac{1}{2} + 2\right)\right|$

$+ (4 + 2 - 4) - \left(\frac{1}{4} + \frac{1}{2} - 2\right)$

$= \left|\frac{-5}{4} - \frac{11}{4}\right| + 2 + \frac{5}{4}$

$= 4 + 2 + \frac{5}{4}$

$= 7\frac{1}{4}$ units^2

d) find points of intersection

$x^2 + 1 = x + 7$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

$\therefore A = \int_{-2}^{3} x + 7 - (x^2 + 1)\, dx$

$= \int_{-2}^{3} x + 7 - x^2 - 1\, dx$

$= \int_{-2}^{3} x - x^2 + 6\, dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3} + 6x\right]_{-2}^{3}$

$= \frac{27}{2} - \frac{34}{3}$

$= 2\frac{1}{6}$ units^2

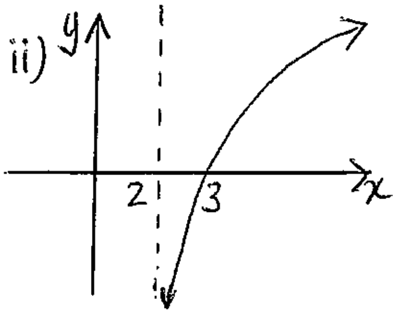
Question 7

a) $y = \log(x-2)$

i) $x - 2 > 0$

$x > 2$

ii)



b) $\sqrt[x]{m} = n^3$

$m^{1/x} = n^3$

$\frac{1}{x}\log m = 3\log n$

$\frac{1}{x} = \frac{3\log n}{\log m}$

$x = \frac{\log m}{3\log n}$

c) $\int e^{3x+2}\,dx$

$= \frac{1}{3}e^{3x+2} + c$

d) $y = e^{x^2}$

$\frac{dy}{dx} = 2xe^{x^2}$

$\frac{d^2y}{dx^2} = 2x.2xe^{x^2} + 2e^{x^2}$

$= 2e^{x^2}(2x^2+1)$

e) $V = \pi\int_a^b y^2\,dx$

$= \pi\int_0^2 (e^x+1)^2\,dx$

$= \pi\int_0^2 e^{2x} + 2e^x + 1\,dx$

$= \pi\left[\frac{1}{2}e^{2x} + 2e^x + x\right]_0^2$

$= \pi\left[\left(\frac{e^4}{2} + 2e^2 + 2\right) - \left(\frac{1}{2} + 2 + 0\right)\right]$

$= \pi\left(\frac{e^4}{2} + 2e^2 - \frac{1}{2}\right)$

f) i) all real x, $x \neq 0$

ii) $y = \frac{1}{x}e^{-x}$

$\frac{dy}{dx} = \frac{-1}{x}e^{-x} - \frac{1}{x^2}e^{-x}$

$= e^{-x}\left(\frac{-1}{x} - \frac{1}{x^2}\right)$

for stationary points, $\frac{dy}{dx} = 0$

ie $e^{-x}\left(\frac{-1}{x} - \frac{1}{x^2}\right) = 0$

$\frac{-1}{x} - \frac{1}{x^2} = 0$

$\frac{-x-1}{x^2} = 0$

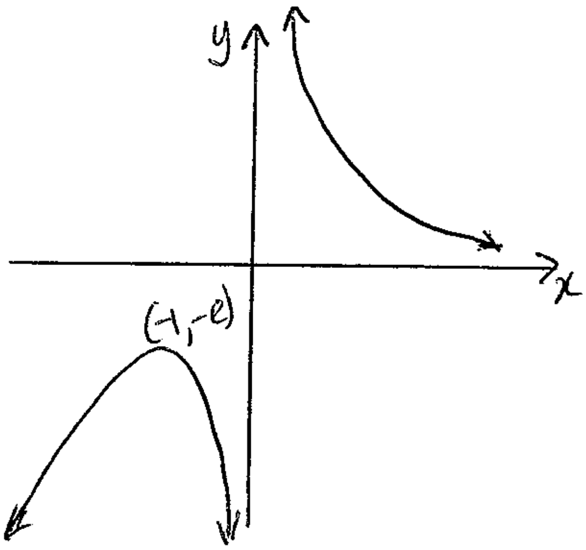
$-x - 1 = 0$

$x = -1$

x	$-3/2$	-1	$-1/2$
$f'(x)$	>0	0	<0

$\therefore$ max at $(-1, -e)$

iii)



Question 8

a) $x^2-(2+k)x+3k+6$

+ve definite when $\Delta < 0$
and $a > 0$

$b^2-4ac<0$

$(2+k)^2-4(3k+6)<0$

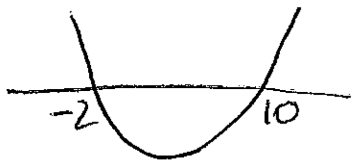
$4+4k+k^2-12k-24<0$

$k^2-8k-20<0$

Solve $k^2-8k-20=0$

$(k-10)(k+2)=0$

$k=+10,-2$



$\therefore -2<k<10$

b) $(8,4)$ $\quad x-2y-1=0$

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{8-8-1}{\sqrt{1+4}}\right|$$

$$=\frac{1}{\sqrt{5}}$$

c) $y=x^3+6x^2-4x+7$

concave down when $\frac{d^2y}{dx^2}<0$

$$\frac{dy}{dx}=3x^2+12x-4$$

$$\frac{d^2y}{dx^2}=6x+12$$

$6x+12<0$

$6x<-12$

$x<-2$

d) i) $A=\frac{1}{4}\times\pi\times 9$

$=\frac{9\pi}{4}$ units^2

ii) $A=\frac{1}{2}bh$

$=\frac{1}{2}\times p\times\sqrt{9-p^2}$

$=\frac{p\sqrt{9-p^2}}{2}$

iii)

$$\frac{dA}{dp} = \frac{p}{2} \cdot \frac{1}{2}(9-p^2)^{-1/2} \cdot -2p + \frac{1}{2}(9-p^2)^{1/2}$$

$$= \frac{-p^2}{2\sqrt{9-p^2}} + \frac{\sqrt{9-p^2}}{2}$$

$$= \frac{-p^2+9-p^2}{2\sqrt{9-p^2}}$$

$$= \frac{9-2p^2}{2\sqrt{9-p^2}}$$

Max when $\frac{dA}{dp} = 0$

$$\frac{9-2p^2}{2\sqrt{9-p^2}} = 0$$

$$9-2p^2 = 0$$

$$2p^2 = 9$$

$$p^2 = \frac{9}{2}$$

$$p = \frac{3}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{2}$$

$f'(1) > 0, \quad f'(2.5) < 0$

$\therefore$ max

$\therefore P\left(\frac{3\sqrt{2}}{2} \sqrt{9-\frac{18}{4}}\right)$

$= \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

iv)

$$A = \frac{1}{2} \times \frac{3\sqrt{2}}{2} \times \frac{3\sqrt{2}}{2}$$

$$= \frac{18}{8}$$

$$= \frac{9}{4}$$

ratio is $\frac{9}{4} : \frac{9\pi}{4}$

$= 1 : \pi$